

# **MATHEMATICS METHODS**

**MAWA Semester 1 (Unit3) Examination 2016**

**Calculator-Assumed**

**Marking Key**

Section Two: Calculator-assumed

(98 Marks)

Question 10

Solution $\frac{dV}{dr} = 2\pi r^2$ $\frac{\delta r}{r} \approx \frac{1}{2\pi r^2} \times \frac{\delta V}{r}$ $= \frac{1}{3} \times \frac{3}{2\pi r^3} \times \delta V$ $= \frac{1}{3} \times \frac{\delta V}{V}$ $= \frac{1}{3} \times \frac{1.5}{100}$ $= 0.005 \times 100 = 0.5\%$		OR	$V = \frac{2}{3} \pi r^3$ $\delta V \approx 2\pi r^2 \delta r$ $\frac{\delta V}{V} = \frac{2\pi r^2}{\frac{2}{3} \pi r^3} \delta r = 3 \frac{\delta r}{r}$ $\therefore \frac{\delta r}{r} = \frac{0.015}{3} = 0.005 \times 100 = 0.5\%$
Marking key/mathematical behaviours			Marks
<ul style="list-style-type: none"> <li>• states the correct volume</li> <li>• uses incremental formula correctly</li> <li>• writes the incremental formula as ratios</li> <li>• calculates the correct percentage change</li> </ul>			1 1 1 1

Question 11(a)

Solution	<table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px; text-align: center;">y</td> <td style="padding: 5px; text-align: center;">1</td> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">3</td> </tr> <tr> <td style="padding: 5px; text-align: center;">P(Y = y)</td> <td style="padding: 5px; text-align: center;"><math>\frac{1}{3}</math></td> <td style="padding: 5px; text-align: center;"><math>\frac{1}{2}</math></td> <td style="padding: 5px; text-align: center;"><math>\frac{1}{6}</math></td> </tr> </table>	y	1	2	3	P(Y = y)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
y	1	2	3						
P(Y = y)	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$						
Marking key/mathematical behaviours	<ul style="list-style-type: none"> <li>• calculates both probabilities correctly</li> </ul>								
	Marks								
	1								

Question 11(b)(i)

Solution	
$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$	
Marking key/mathematical behaviours	
<ul style="list-style-type: none"> <li>• calculates correct probability</li> </ul>	
	Marks
	1

**Question 11(b)(ii)**

Solution $\frac{1}{2}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>calculates correct probability</li> </ul>	1

**Question 11(b)(iii)**

Solution $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>calculates correct probability</li> </ul>	1

**Question 11(b)(iv)**

Solution $\frac{1}{3} \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{6} = \frac{7}{18}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>indicates both faces being 1, 2 or 3</li> <li>calculates correct probability</li> </ul>	1 1

**Question 11(b)(v)**

Solution Possible pairings are 1 3 or 3 1 or 2 2 $\frac{1}{3} \times \frac{1}{6} \times 2 + \frac{1}{2} \times \frac{1}{2} = \frac{13}{36}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>gives correct pairings including both possibilities for 1 and 3</li> <li>calculates correct probability</li> </ul>	1, 1 1

**Question 12(a)**

Solution $\frac{dy}{dx} = -4axe^{x^2}$ $0 = -4axe^{x^2}$ $x = 0$ when $x = 0$ , $y = a - 2ae^0$ $y = -a$ stationary point at $(0, -a)$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>determines the derivative using the chain rule</li> </ul>	1
<ul style="list-style-type: none"> <li>equates to zero and solves</li> </ul>	1
<ul style="list-style-type: none"> <li>substitutes to determine y-coordinate</li> </ul>	1

**Question 12(b)**

Solution $\frac{d^2y}{dx^2} = -4ax(2xe^{x^2}) - 4ae^{x^2}$ $\left. \frac{d^2y}{dx^2} \right _{x=0} = -4a$ Since $a$ is a positive constant the second derivative is negative. It is a maximum	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>determines the first and second parts of the second derivative using the product rule and chain rule</li> </ul>	1,1
<ul style="list-style-type: none"> <li>determines the value of the second derivative when <math>x=0</math></li> </ul>	1
<ul style="list-style-type: none"> <li>states the nature of the stationary point</li> </ul>	1

**Question 13(a)**

Solution $\frac{dy}{dx} = x\cos(x) + \sin(x)$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>correctly differentiates using the product rule</li> </ul>	1,1

**Question 13(b)**

Solution	
$\int \frac{dy}{dx} = \int x \cos(x) + \sin(x) dx$ $x \sin(x) = \int x \cos(x) dx + \int \sin(x) dx$ $\int x \cos(x) dx = x \sin(x) - \int \sin(x) dx$ $= x \sin(x) + \cos(x) + c$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>integrates both sides of the derivative obtained in part (a)</li> <li>replaces the LHS by <math>y</math></li> <li>rearranges correctly</li> <li>integrates <math>\sin(x)</math> correctly</li> </ul>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

**Question 14(a)**

Solution							
$x$	4	6	8	9	11	14	
$P(X = x)$	0.16	<b>0.32</b>	<b>0.16</b>	<b>0.16</b>	<b>0.16</b>	0.04	
Marking key/mathematical behaviours							Marks
<ul style="list-style-type: none"> <li>calculates correct probability for each score</li> </ul>							1, 1, 1, 1

**Question 14(b)**

Solution							
$y$	-6	-4	-2	-1	1	4	
$P(Y = y)$	0.16	<b>0.32</b>	<b>0.16</b>	<b>0.16</b>	<b>0.16</b>	0.04	
Marking key/mathematical behaviours							Marks
<ul style="list-style-type: none"> <li>correctly completes distribution table</li> </ul>							1, 1, 1

**Question 14(c)**

Solution	
<p>The sum of <math>y \times P(Y = y) = -2.40</math> cents</p> <p>For 50 games = <math>-240</math> cents which is a loss of \$2.40</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>correctly calculates expected value</li> <li>multiplies by 50</li> <li>states that it is a loss</li> </ul>	<p>1</p> <p>1</p> <p>1</p>

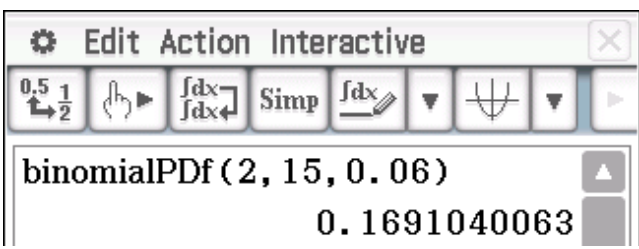
**Question 15**

<p>Solution</p> $\frac{d}{dx} \int_a^x (f(t) + e^t) dt - 2 \int_0^x \frac{d}{dt} (f(t) + e^{2t}) dt = 2$ $f(x) + e^x - 2(f(x) + e^{2x} - f(0) - 1) = 2$ $-f(x) + e^x - 2e^{2x} + 4 = 2$ $f(x) = 2 + e^x - 2e^{2x}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>• applies the fundamental theorem to first integral</li> <li>• evaluates second integral</li> <li>• expands brackets correctly</li> <li>• substitutes <math>f(0)</math></li> <li>• rearranges for <math>f(x)</math> correctly</li> </ul>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>

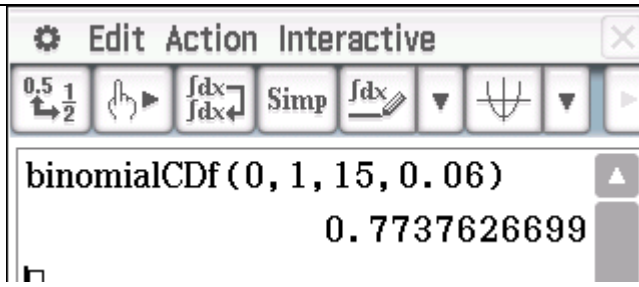
**Question 16**

<p>Solution</p> $\int_0^k (\sqrt{k} - \sqrt{x})^2 dx$ $= \int_0^k k - 2\sqrt{k}x^{\frac{1}{2}} + x dx$ $= \left[ kx - \frac{4}{3}\sqrt{k}x^{\frac{3}{2}} + \frac{x^2}{2} \right]_0^k$ $= k^2 - \frac{4}{3}k^{\frac{1}{2}}k^{\frac{3}{2}} + \frac{k^2}{2} - 0$ $= \frac{k^2}{6}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>• correctly expands brackets</li> <li>• correctly integrates</li> <li>• correctly substitutes limits</li> <li>• correctly simplifies</li> </ul>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

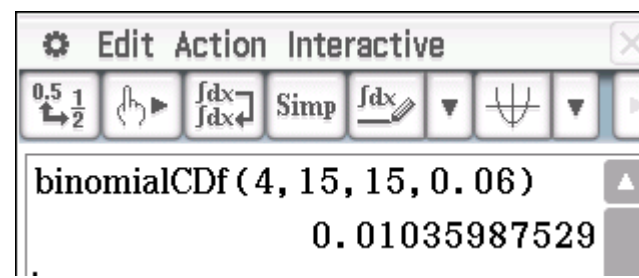
**Question 17(a)**

Solution		
Marking key/mathematical behaviours		Marks
<ul style="list-style-type: none"> <li>determines correct probability</li> </ul>		1

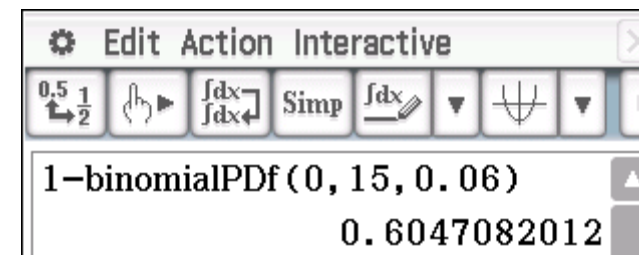
**Question 17(b)**

Solution		
Marking key/mathematical behaviours		Marks
<ul style="list-style-type: none"> <li>determines correct probability</li> </ul>		1

**Question 17(c)**

Solution		
Marking key/mathematical behaviours		Marks
<ul style="list-style-type: none"> <li>determines correct probability</li> </ul>		1

**Question 17(d)**

Solution		
P(at least 1)=1 – P(0)		
Marking key/mathematical behaviours		Marks
<ul style="list-style-type: none"> <li>correctly uses complementary event</li> <li>determines correct probability</li> </ul>		1 1

**Question 17(e)**

<p>Solution Using <math>P(\text{at least } 1) = 1 - P(0)</math> and testing <math>n = 11, 12, 13</math>. Largest sample is 12.</p> <p>OR using solve</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>• correctly uses complementary event and tests 11, 12, 13</li> <li>• determines correct sample size</li> </ul>	<p>1,1</p> <p>1</p>

**Question 18(a)**

<p>Solution In triangle, height = <math>2 \cos \theta</math> and base = <math>2 \sin \theta</math>  <math>V = \text{area of trapezium} \times 8</math>  <math>= \frac{2 \cos \theta}{2} \times (2 + 2 + 2 \times 2 \sin \theta) \times 8</math>  <math>= \cos \theta \times (4 + 4 \sin \theta) \times 8</math>  <math>= 32 \cos \theta (1 + \sin \theta)</math></p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>• identifies height and base of triangle</li> <li>• uses suitable formula for area of base</li> <li>• simplifies and factorises result</li> </ul>	<p>1</p> <p>1</p> <p>1</p>



**Question 18(b)**

Solution	
$\text{solve} \left( 10 = 32 \cdot \cos(\theta) \cdot (1 + \sin(\theta)), \theta, 0, 0, \frac{\pi}{2} \right)$ $\{\theta = 1.412913449\}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>recognises to equate the volume equation to 10</li> </ul>	1
<ul style="list-style-type: none"> <li>solves for theta</li> </ul>	1

**Question 18(c)**

Solution	
$V'(\theta) = -32 \sin \theta (1 + \sin \theta) + 32 \cos \theta \cos \theta$	
For max: $V'(\theta) = 0 \Rightarrow \theta = 0.52$	
$V''(0.52) = -83.14 \Rightarrow$ maximum	
$V(0.52) = 41.57 \text{ m}^3$	
Maximum capacity is 41570 kL	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>determines the first part of the derivative using the product rule</li> </ul>	1
<ul style="list-style-type: none"> <li>determines the second part of the derivative using the product rule</li> </ul>	1
<ul style="list-style-type: none"> <li>equates derivative to zero and solves for theta</li> </ul>	1
<ul style="list-style-type: none"> <li>justifies maximum</li> </ul>	1
<ul style="list-style-type: none"> <li>determines the volume</li> </ul>	1
<ul style="list-style-type: none"> <li>states the capacity</li> </ul>	1

Question 19(a)

Solution

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	0.5	<b>0.54</b>	<b>0.67</b>	1

Rectangle	$0 - \frac{\pi}{6}$	$\frac{\pi}{6} - \frac{\pi}{3}$	$\frac{\pi}{3} - \frac{\pi}{2}$	Total
Lower rectangle area	0.26	<b>0.28</b>	<b>0.35</b>	<b>0.89</b>
Upper rectangle area	0.28	<b>0.35</b>	<b>0.52</b>	<b>1.15</b>
			Mean	<b>1.02</b>

Marking key/mathematical behaviours

Marks

- correctly calculates the values of  $y$
- correctly calculates areas of upper and lower rectangles
- correctly calculates totals
- correctly calculates mean

1,1  
1  
1  
1

Question 19(b)

Solution

$$\begin{aligned} \frac{d}{dx} \left( \frac{\sin x}{1 + \cos x} \right) &= \frac{(1 + \cos x)(\cos x) - \sin x(-\sin x)}{(1 + \cos x)^2} \\ &= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} \\ &= \frac{(\cos x + 1)}{(1 + \cos x)^2} \\ &= \frac{1}{1 + \cos x} \end{aligned}$$

Marking key/mathematical behaviours

Marks

- correctly differentiates using the quotient rule
- expands brackets and simplifies
- uses the given information correctly and obtains solution

1  
1  
1

**Question 19(c)**

Solution	
$\text{Area} = \int_0^{\frac{\pi}{3}} f(x) dx = \left[ \frac{\sin x}{1 + \cos x} \right]_0^{\frac{\pi}{3}}$ $= \frac{2}{\sqrt{3}}$ $= \frac{2}{3}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>correctly uses part (c) for the integral</li> <li>evaluates the integral</li> <li>simplifies solution</li> </ul>	<p>1</p> <p>1</p> <p>1</p>

**Question 20(a)**

Solution	
<pre>solve(0=2*e<sup>2*t</sup>-10, t)       {t=0.8047189562}</pre>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>recognises that the particle is at rest when <math>v = 0</math></li> <li>Solves for <math>t</math>.</li> </ul>	<p>1</p> <p>1</p>

**Question 20(b)**

Solution	
$a(t) = \frac{dv}{dt}$ $= 4e^{2t}$ $a(0) = 4(1)$ $= 4 \text{ m/s}^2$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>determines the derivative of the velocity function</li> <li>determines the acceleration when <math>t = 0</math>.</li> </ul>	<p>1</p> <p>1</p>

**Question 20 (c)**

<p>Solution</p> $x(t) = \int (2e^{2t} - 10)dt$ $= e^{2t} - 10t + c$ $3 = e^0 + c$ $c = 2$ $x(2) = e^4 - 20 + 2$ $= 36.60m$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>integrates <math>v(t)</math> to obtain general rule for <math>x(t)</math></li> <li>uses the initial conditions to determine <math>c</math></li> <li>calculates the displacement at <math>t=2</math></li> </ul>	<p>1</p> <p>1</p> <p>1</p>

**Question 20 (d)**

<p>Solution</p> $x(t) = \int_0^4  2e^{2t} - 10  dt$ $= 2948.05 \text{ m}$ <p>OR</p> $x(0) = 3$ $x(0.80) = -1.05$ $x(4) = 2942.96$ $DIST = 4.05 + 1.05 + 2942.96$ $= 2948.05 \text{ m}$	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>recognises the need for absolute value</li> <li>identifies the limits as 0 and 4</li> <li>determines the distance travelled</li> </ul>	<p>1</p> <p>1</p> <p>1</p>

**Question 21 (a)**

Solution	
<pre>Define P(t)=1500*e<sup>0.07*t</sup>                                 done P(3)                                 1850.51709</pre>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>writes the function for the population</li> <li>determines the population when <math>t=3</math></li> </ul>	<p>1</p> <p>1</p>

**Question 21 (b)**

Solution	
<pre>solve(P(t)=2000, t)                                 {t=4.109743892}</pre> <p>During 2014</p>	
Marking key/mathematical behaviours	Mark
<ul style="list-style-type: none"> <li>determines the value of <math>t</math> when <math>P = 2000</math></li> <li>states the year this occurs</li> </ul>	<p>1</p> <p>1</p>

**Question 21(c)**

Solution	
<pre>P(6)                                 2282.942333 Define Q(t)=P(6)*e<sup>-0.05*t</sup>                                 done solve(1500=Q(t), t)                                 {t=8.4}</pre> <p>During May 2024</p>	
Marking key/mathematical behaviours	Marks
<ul style="list-style-type: none"> <li>determines the population at the start of 2016</li> <li>states an equation for the new population</li> <li>equates this new equation to 1500 and solves for <math>t</math></li> <li>states the month and year corresponding to this value of <math>t</math></li> </ul>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>

Question 22

Solution

$$\text{Height of triangle } OPR = ah^2 + bh$$

$$\text{Area under curve from 0 to } h = \int_0^h ax^2 + bx \, dx = \left[ \frac{ax^3}{3} + \frac{bx^2}{2} \right]_0^h = \frac{ah^3}{3} + \frac{bh^2}{2}$$

$$\text{Equation of line } OQ : \frac{d}{dx}(ax^2 + bx) \Big|_{x=0} = b \therefore \text{equation of line } OQ \text{ is } y = bx$$

$$\text{Area of triangle } OPR : \frac{h}{2}(ah^2 + bh) = \frac{ah^3}{2} + \frac{bh^2}{2}$$

$$\text{Area of triangle } OQR : \frac{h}{2} \times bh = \frac{bh^2}{2}$$

$$\text{Area of region } A : \frac{ah^3}{2} + \frac{bh^2}{2} - \left( \frac{ah^3}{3} + \frac{bh^2}{2} \right) = \frac{ah^3}{6}$$

$$\text{Area of region } B : \frac{ah^3}{3} + \frac{bh^2}{2} - \frac{bh^2}{2} = \frac{ah^3}{3}$$

$$\text{Ratio of region } A \text{ to region } B : \frac{\frac{ah^3}{6}}{\frac{ah^3}{3}} = 1 : 2$$

Marking key/mathematical behaviours	Marks
• determines height of triangle $OPR$ in terms of $h$	1
• determines area under curve	1
• determines equation of line $OQ$	1
• determines areas of triangles $OPR$ and $OQR$	1
• determines area of region $A$ and $B$	1
• calculates ratio of region $A$ to region $B$	1

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