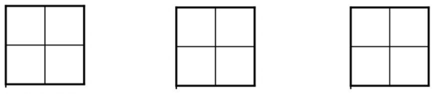
An example of an investigative activity

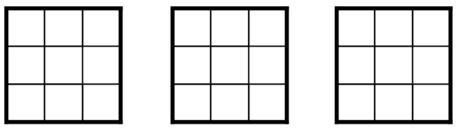
The initial problem is based on a 2 by 2 square grid.



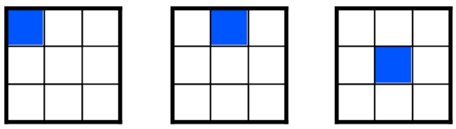
The problem is to show all of the different ways in which two of the four squares can be coloured (blue) The students are told that two colourings are NOT different if one can be matched with the other by rotation. If they are not different, they are called duplicates. Reflections (flips) are not accepted). The two colourings below are duplicates of each other (by rotation of 90 degrees)

Here are the two solutions to the problem.

The problem is then extended to showing all the different ways in which **one** square can be coloured on a 3 by 3 square grid

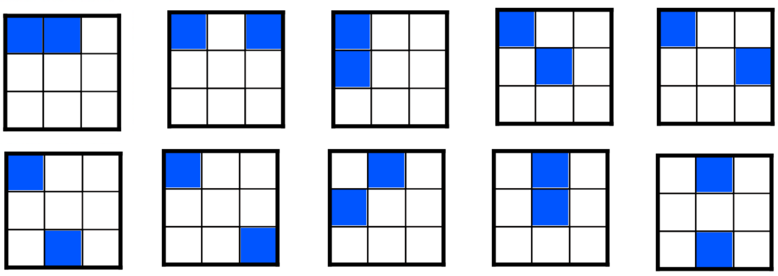


The three solutions to this problem are shown below



I should add that recognizing and avoiding duplicates is a very useful exercise in developing mastery in spatial relationships.

The next extension is to ask the students to show all of the different ways of colouring **two** of the squares on a 3 by 3 square grid. For mathematically-able 11 year-old students this turns out to be quite a difficult problem. Typically only about 4 or 5 students in a class of 30 will get the correct result at their first attempt. The ten solutions shown below were obtained using a systematic approach to the colourings.

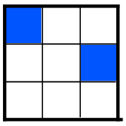


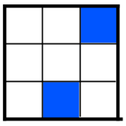
Calculation of the number of 2-square colourings on a 3 by 3 square grid

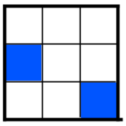
Solution:

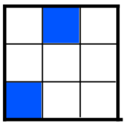
1 Number the 9 squares as shown below

1	2	3
4	5	6
7	8	9

2 Then the colouring  is associated with {1,6}.

The duplicate  is associated with {3,8}.

The duplicate  is associated with {4,9}.

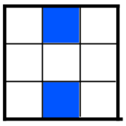
The duplicate  is associated with {2,7}.

In this way it should be clear that each of the ten colourings is associated with a 2-member subset of {1,2,3,4,5,6,7,8,9}.

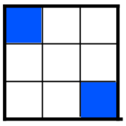
3 The number of 2-member subsets (ie pairs) that can be obtained from {1,2,3,4,5,6,7,8,9} is $(9 \times 8) \div 2 = 36$. If every colouring was like that shown in 2 above, then every colouring would have 4 corresponding subsets, and the number of different colourings would be $36 \div 4 = 9$.
BUT WE ALREADY KNOW THERE ARE TEN DIFFERENT COLOURINGS !

4 RESOLVING THE CONTRADICTION

Not all colourings produce four subsets when rotated through 360 degrees.



Produces only two subsets
{2,8} and {4,6}



Produces only two subsets
{1,9} and {3,7}

Of the total of 36 subsets, the above 2 “special” colourings account for 4 subsets. The remaining 32 subsets are associated, in batches of 4, with “ordinary” colourings.

There must therefore be 8 “ordinary” colourings. ***There are thus 2 “special” colourings and 8 “ordinary’ colourings, making a total of 10 different colourings.*** This confirms the result that had already been found by systematically doing the colourings

I hope that this example gives readers some appreciation o of the thinking capabilities of mathematically-able 11 year-old students. I reiterate that we owe it to these students to help them develop and extend their special capabilities

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