Three Types of Task
- Extended Investigations
- In-class Investigations
- Investigative Questions

18 Assessment Tasks

Includes solutions with marking keys showing mathematical behaviours

Written by teachers for teachers

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Australian Curriculum

The Mathematical Association of Western Australia Inc.

MAΣWA

Free Sample investigation (in-class)

INVE… …TION A… …MENT TASKS YEAR 11 MATHEMATICAL METHODS

Contents

TASK 11

EXPONENTIAL FUNCTIONS

In-class investigation

Unit 2

Topic 2.1: Exponential functions

Course-related information

The concepts and skills developed in this investigation relate to the following content descriptors in the Australian curriculum Mathematical Methods syllabus:

use function notation; determine domain and range; recognise independent and dependent variables (ACMMM023)

review indices (including fractional indices) and the index laws (ACMMM061)

establish and use the algebraic properties of exponential functions (ACMMM064)

recognise the qualitative features of the graph of $y = a^x$ $(a > 0)$, including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x+c}$) (ACMMM065)

Background information

Students should have reviewed their knowledge of indices and the index laws before sitting this assessment. They should also be familiar with, and be able to use function notation. In the investigation students are provided with the opportunity to connect their existing knowledge of the index laws with the graphs of exponential functions and the transformations of such functions.

Task conditions

This task consists of an in-class investigation for which students might need 40 – 55 minutes to complete. Students need to be familiar with tabulation and graphing using technology and it is expected that access to such technology will be available during the investigation.

Exponential functions

In-class investigation (52 marks)

This investigation focuses on exponential functions which have the general formula

$f(x) = a^x$ and where $a > 0$

Question 1 (21 marks)

(a) Complete the tables provided for each of the given functions, rounding your answers to one decimal place. (5)

(i)

x	-2	-1	0	1	2	3
$f(x)=2^x$						

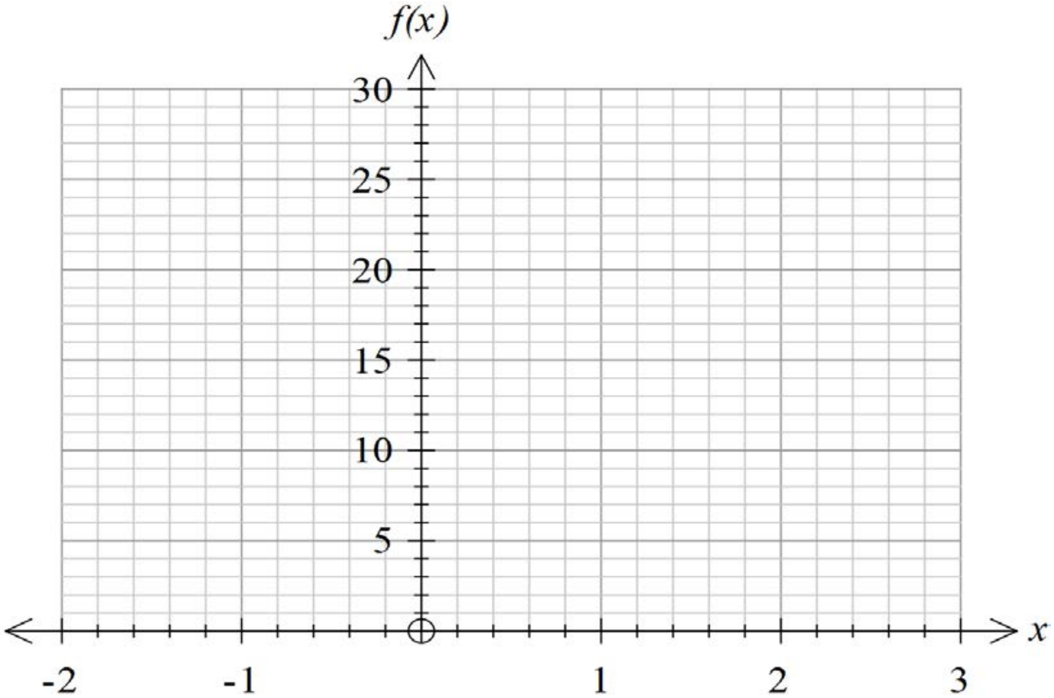
(ii)

x	-2	-1	0	1	2	3
$f(x)=3^x$						

(iii)

x	-2	-1	0	1	2	3
$f(x)=4^x$						

(b) Sketch the three functions on the grid provided. (5)



(c) State the domain for the function $f(x)$. With reference to the domain, describe and justify the values $f(x)$ can/cannot take. (2)

(d) State the y-intercept for each of the three functions. (2)

(e) State the general rule relating the x value at the y-intercept to the value of $f(x)$. (1)

(f) Considering both positive and negative values of x, describe the changes to the graph of $f(x) = a^x$ as a increases ($a > 1$). (2)

(g) Given $f(x)=2^x$ and $g(x) = 2^{x+1}$

(i) state the value of $f(3)$

(ii) state the value of $g(3)$

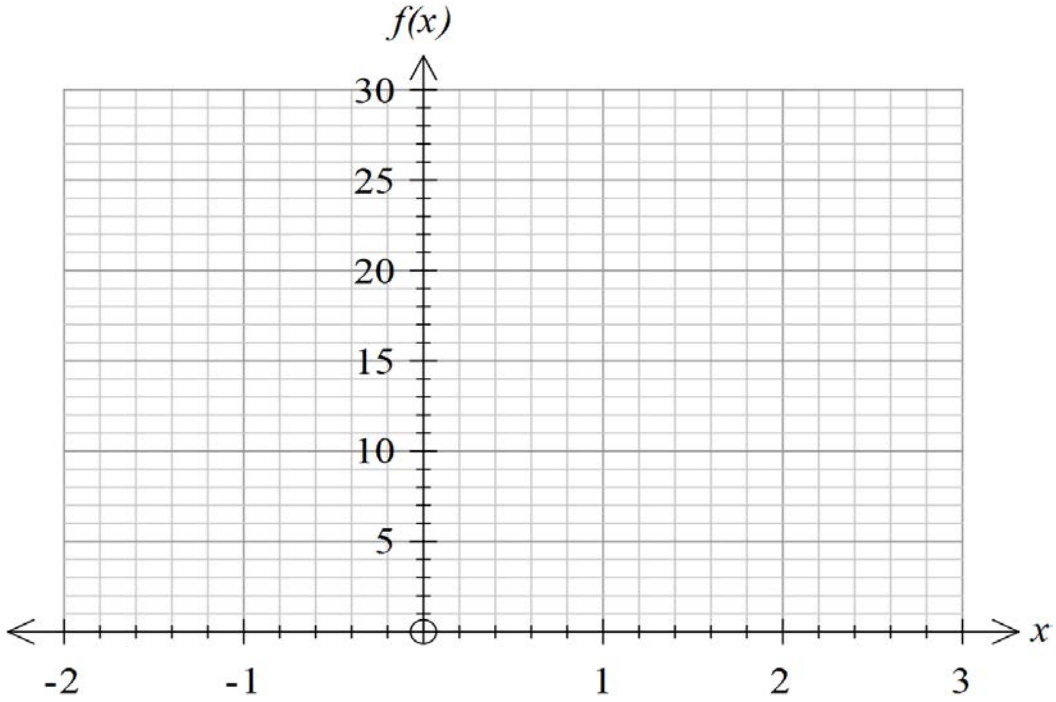
(iii) describe the value of $g(x)$ in terms of the value of $f(x)$ (3)

(h) Given $h(x)=m^x$ and $k(x) = m^{x+1}$ describe $k(x)$ in terms of $h(x)$without any reference to m. (1)

Question 2 **(11 marks)**

(a) On the grid provided, sketch and label each of the following functions. (5)

$f(x) = 2^x$
$g(x) = 2^{x+1}$
$h(x) = 2^{x+2}$
$j(x) = 2^{x+3}$



(b) State the y-intercepts for each of these functions. (2)

(i) $f(x) = 2^x$

(ii) $g(x) = 2^{x+1}$

(iii) $h(x) = 2^{x+2}$

(iv) $j(x) = 2^{x+3}$

(c) The graph of $y = 2^{x+c}$ may be considered as a translation or a dilation of the graph of $y = 2^x$. Describe the graph of $y = 2^{x+c}$ as (4)

(i) a translation of the graph of $y = 2^x$

(ii) a dilation of the graph of $y = 2^x$

Question 3 **(10 marks)**

(a) For the functions below enter the values and express any numbers which are not whole numbers, as fractions. (4)

(i)

x	-2	-1	0	1	2	3
$f(x)=5^{x}$						

(ii)

x	-2	-1	0	1	2	3
$f(x)=10^{x}$						

(iii)

x	-2	-1	0	1	2	3
$f(x)=5^{-x}$						

(iv)

x	-2	-1	0	1	2	3
$f(x)=10^{-x}$						

(b) Describe the similarity in the tables for $f(x)=10^{x}$ and $f(x)=10^{-x}$. (2)

(c) What is the rule linking 10^{-x} and 10^{x} ? (1)

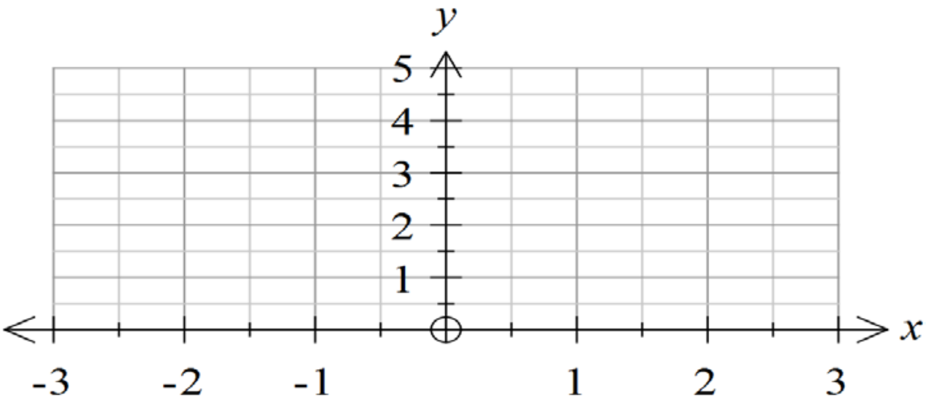
(d) State the rule linking h^{n} and h^{-n} . For what values of n, if any, does the rule not apply? (2)

(e) If $h^{n} = 216$, give the value of h^{-n}. (1)

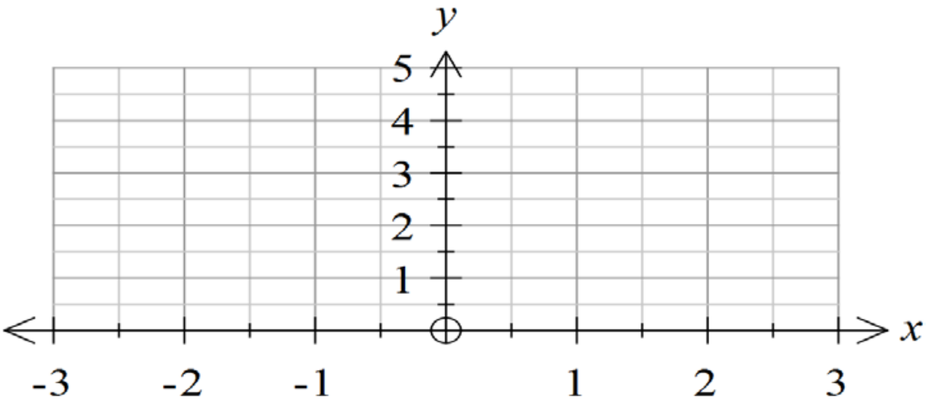
Question 4 **(20 marks)**

(a) Sketch the following pairs of graphs on the axes provided. (9)

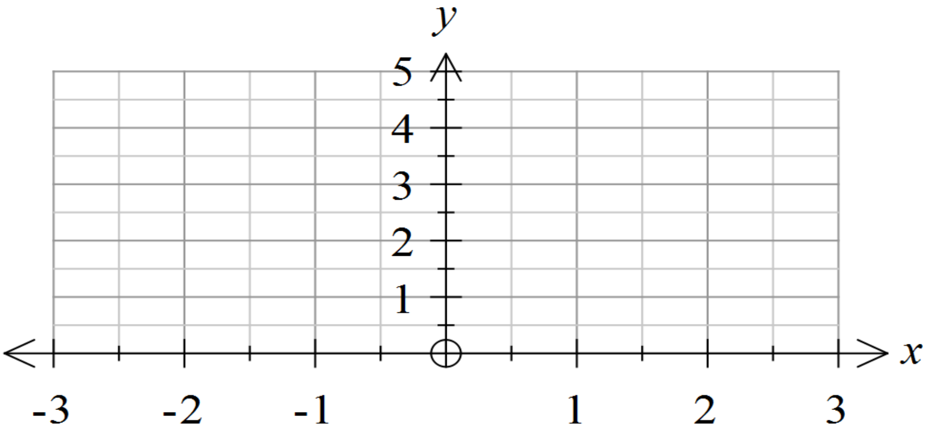
(i) $y = \left(\frac{3}{4}\right)^x$ and $y = \left(\frac{4}{3}\right)^x$



(ii) $y = \left(\frac{3}{5}\right)^x$ and $y = \left(\frac{5}{3}\right)^x$



(iii) $y = \left(\frac{2}{3}\right)^x$ and $y = \left(\frac{3}{2}\right)^x$



(b) Complete the table below by writing expressions for $\left(\frac{b}{a}\right)^x$ derived from the values for $\left(\frac{a}{b}\right)^x$ given in the table. (7)

x	-2	-1	0	2	3
$\left(\frac{a}{b}\right)^x$	m	n	p	$\frac{d}{e}$	$\frac{g}{k}$
$\left(\frac{b}{a}\right)^x$					

(i) Given two expressions for m in terms of a and b.

(ii) State the value of p.

(c) The graph of $y=\left(\frac{3}{4}\right)^x$ is a transformation of $y=\left(\frac{4}{3}\right)^x$ (4)

(i) Describe this transformation.

(ii) Explain how the relationship between the two expressions accounts for the transformation that occurs.

Exponential functions

In-class investigation
Solutions

Question 1

(a) (i)

x	-2	-1	0	1	2	3
$f(x)=2^x$	0.3	0.5	1	2	4	8

(ii)

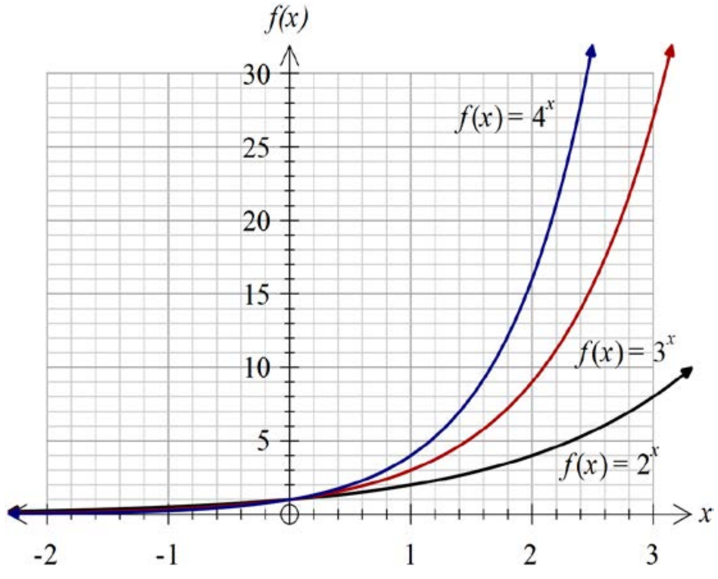
x	-2	-1	0	1	2	3
$f(x)=3^x$	0.1	0.3	1	3	9	27

(iii)

x	-2	-1	0	1	2	3
$f(x)=4^x$	0.1	0.3	1	4	16	64

Marking key/mathematical behaviours	Marks
• Provides accurate values in all cases	1
• Determines values for each of the three functions	3
• Correctly rounds to one decimal place	1

(b)



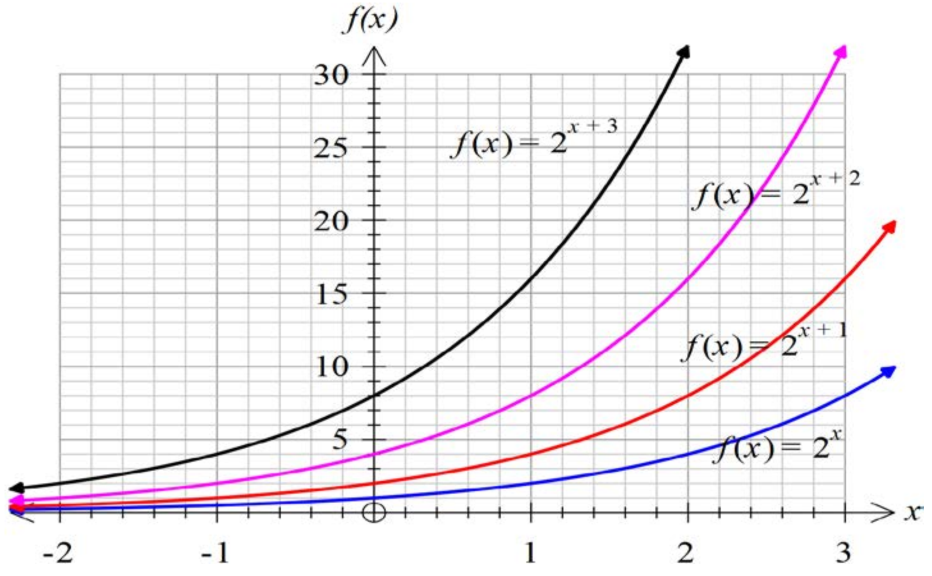
Marking key/mathematical behaviours	Marks
• Graphs drawn with equivalent y-intercepts	1
• Draws graphs of the correct shapes	1
• Labels each graph	1
• Correctly positions graphs relative to each other	1
• Graphs drawn to approach but not intersect x-axis	1

Question 1 (cont'd)

	Solution	Marking key/mathematical behaviours	Marks
(c)	$f(x) > 0$ There are no values of x for which $f(x) = 0$ and if a >0 the function cannot be negative	• Identifies range for $f(x)$ • Explains limits on function range	1 1
(d)	(0,1) for all functions	• States the y-intercept • Applies y-intercept for all functions	1 1
(e)	$a^0=1$	• Provides rule for x=0	1
(f)	As a increases the graph of $f(x)=a^x$ gets steeper and closer to the y-axis while x>0. For x <0 the graph gets closer to the x -axis.	• Describes increasing gradient for x>0 • Describes decreasing gradient for x <0	1 1
(g)	(i) 8 (ii) 16 (iii) $g(x) = 2\,f(x)$	• Determines value for $f(x)$ • Determines value for $g(x)$ • Relates $g(x)$ and $f(x)$	1 1 1
(h)	$k(x) = h(x+1)$	• Relates $g(x)$ and $f(x)$	1

Question 2

(a)



Marking key/mathematical behaviours	Marks
• Labels graphs with functions	1
• Places graphs in correct relative positions	1
• Identifies correct y-intercepts on graphs	1
• Draws graphs through one other correct coordinate where $x \geq 1$	1
• Shapes graphs correctly	1

Question 2 (cont'd)

	Solution	Marking key/mathematical behaviours	Marks
(b)	(i) (0, 1) (ii) (0, 2) (iii) (0, 4) (iv) (0, 8)	• Identifies all intercepts	2
(c)	(i) $y = 2^{x+c}$ is a horizontal translation of $y = 2^x$, c units to the left. (ii) $y = 2^{x+c}$ is a dilation of $y = 2^x$ parallel to the y axis, scale factor of 2^c.	• Identifies a horizontal translation • Identifies c units to the left • Identifies dilation parallel to y axis • Identifies scale factor	1 1 1 1

Question 3

(a)

(i)

x	-2	-1	0	1	2	3
$f(x)=5^x$	$\frac{1}{25}$	$\frac{1}{5}$	1	5	25	125

(ii)

x	-2	-1	0	1	2	3
$f(x)=10^x$	$\frac{1}{100}$	$\frac{1}{10}$	1	10	100	1000

(iii)

x	-2	-1	0	1	2	3
$f(x)=5^{-x}$	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$

(iv)

x	-2	-1	0	1	2	3
$f(x)=10^{-x}$	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

Marking key/mathematical behaviours	Marks
• Correctly enters integer values to tables • Correctly enters fractional values to tables	2 2

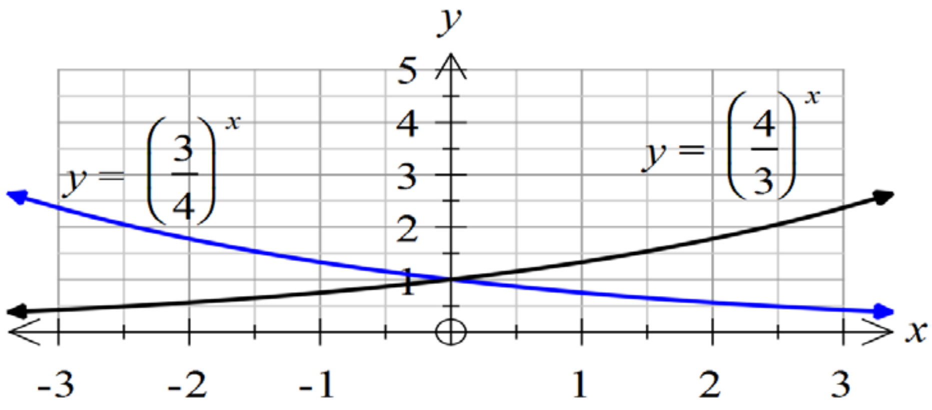
Question 3 (cont'd)

	Solution	Marking key/mathematical behaviours	Marks
(b)	The tables are symmetrical in that $f(x)=10^x$ equals $f(x)=10^{-x}$ when the x value is multiplied by -1.	• Identifies equal values • Links equivalence to changing x values	1 1
(c)	$10^{-x}=\frac{1}{10^x}$	• Identifies relevant rule	1
(d)	$h^n=\frac{1}{h^n}$ Applies for all n	• Generalises rule • Identifies values of n for which the rule applies	1 1
(e)	$\frac{1}{216}$	• Applies identified rule	1

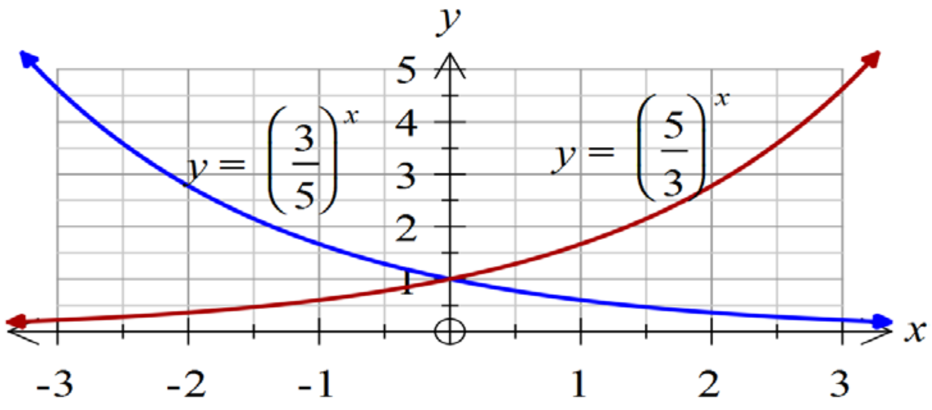
Question 4

(a)

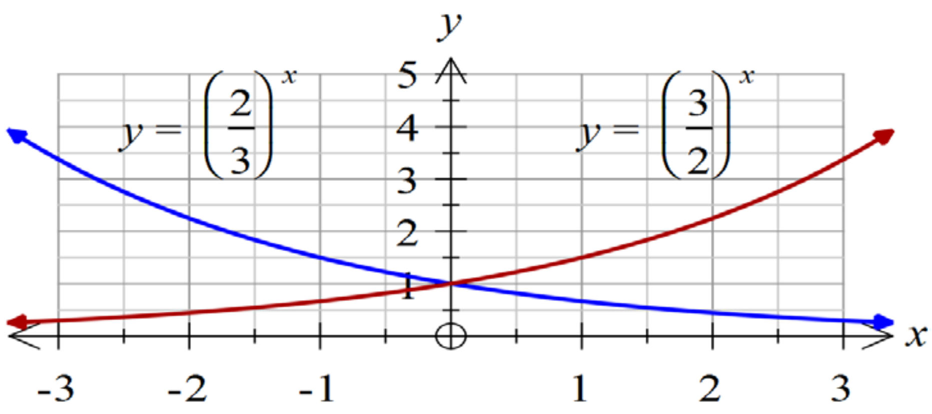
(i)



(ii)



(iii)



Marking key/mathematical behaviours	Marks
• Correctly indicates y-intercepts on the graphs	1
• Correctly indicates x-axis as an asymptote	1
• Draws each pair of graphs symmetrically	1
• Labels graphs throughout	1
• Draws graphs with the correct shape	1
• Positions graphs correctly with at least one point other than the y-intercept in correct position	4

Question 4 (cont'd)

(b)

x	-2	-1	0	2	3
$\left(\frac{a}{b}\right)^x$	m	n	p	$\frac{d}{e}$	$\frac{g}{k}$
$\left(\frac{b}{a}\right)^x$	$\frac{1}{m}$	$\frac{1}{n}$	$\frac{1}{p}$	$\frac{e}{d}$	$\frac{k}{g}$

(i) $m=\left(\frac{a}{b}\right)^{-2}$, $m=\left(\frac{b}{a}\right)^{2}$

(ii) $p=1$

Marking key/mathematical behaviours	Marks
• Completes the table for x = -2, -1 and 0 • Completes the table for x = 2 and 3 • Provides two accurate expressions for m • Identifies p as 1	2 2 2 1

	Solution	Marking key/mathematical behaviours	Marks
(c)	(i) Reflection over the y-axis	• Identifies the reflection • Identifies the line of reflection	1 1
	(ii) Since $\left(\frac{w}{k}\right)^x=\left(\frac{k}{w}\right)^{-x}$ i.e. $\left(\frac{3}{4}\right)^x=\left(\frac{4}{3}\right)^{-x}$ for opposite values of x, the functions have the same value. The opposite values of x position the points equidistant from the origin and the equal values of y position the points equidistant from the x -axis. Hence the symmetry.	• Explains the symmetry	2